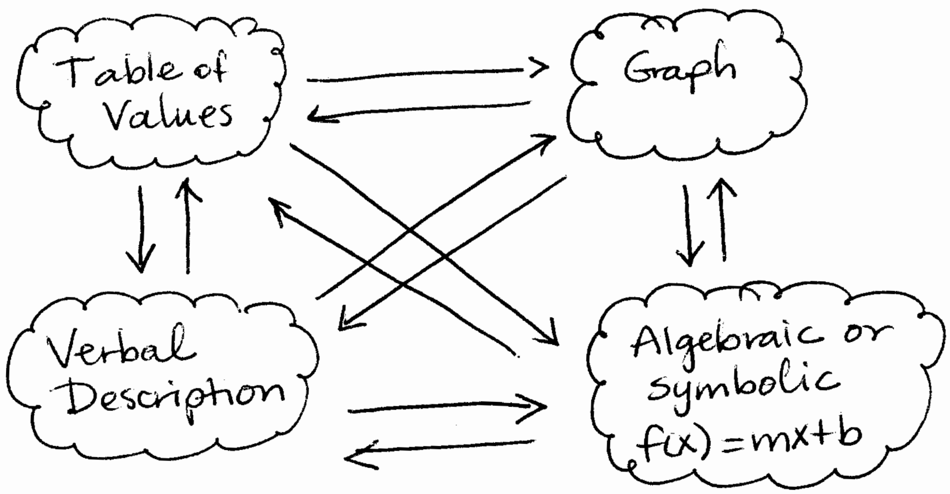
Math 72: 2.2 Linear Functions & 2.3 Slope of a Line
Rockswold textbook

Objectives

1) Identify a linear function (or not) when given
   a) a table of values
   b) a graph
   c) a verbal description (sentence)
   d) algebraically (or "symbolically")

2) Given one of these representations, find the others



3) Find the midpoint between two points
   a) on a number line
   b) on an x-y plane

* 4) Calculate the slope of a line when given two points (ordered pairs) on the line, using the slope formula.

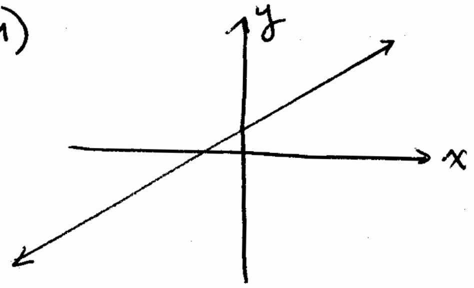
* 5) Interpret the slope of a line as a rate of change, using
   - units of y
   - units of x
   - "increase", "decrease", or "stays the same"
   - "per"

* 6) Slope-intercept form of the equation of a line.

Identifying a linear function.

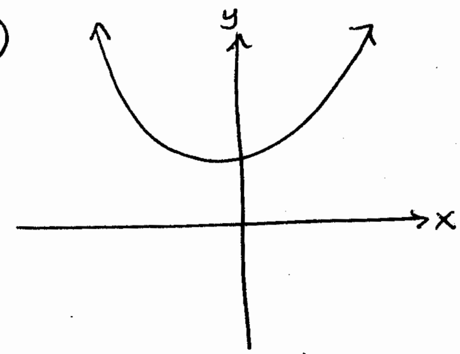
① Does this graph represent a linear function?

a)



$\boxed{\text{yes}}$: This graph is a line

b)



$\boxed{\text{no}}$ This graph is a curve.

② Write a sentence describing this linear function, then an algebraic formula.

"Over a 5-hour period, the A/C lowers the temperature from an initial (starting) temperature of 80°F by 2°F for each elapsed hour x."

"lowers" ⇒ change in temperature is negative

"lowers 2°F" ⇒ -2

"for each hour x" ⇒ $-2x$

"initial temperature" ⇒ start at 80°F

a) sentence (verbal representation)

"Take 80°F and subtract $2x$"

or "Multiply -2 by x and add 80"

b) $\boxed{f(x) = 80 - 2x}$

or $\boxed{f(x) = -2x + 80}$

Recall from Math 45:

Slope-intercept form $y = mx + b$

Linear function $f(x) = mx + b$

(replace y by $f(x)$)

$f(0) = b$

③ Is this table a linear function?

x	0	1	2	3	4	5
$f(x)$	80	78	76	74	72	70

as x increases by 1 unit
y (or $f(x)$) decreases by 2 units } every time!

The fact that the rate of change is constant, always down 2 y-units for each 1-x-unit, is the important characteristic of a linear function.

> Linear functions $f(x) = mx + b$
> have a constant rate of change. This means
>
> when x increases by 1 unit
> y changes ((+)increase/(−)decrease) by m units.

④ Is this function linear or non-linear?

a) $f(x) = 4 - 3x$

b) $f(x) = 8$

c) $f(x) = 2x^2 + 8$

a) $f(x) = 4 - 3x$ can be rewritten as $f(x) = -3x + 4$
This is $\boxed{\text{linear}}$

b) $f(x) = 8$ can be rewritten as $f(x) = 0x + 8$
This is $\boxed{\text{linear}}$

This rate of change is 0 units per unit in x
This function has both a constant rate of change (0) and a constant value (8).
It is called a constant function.

> Constant Functions $f(x) = b$

c) $f(x) = 2x^2 + 8$ has exponent 2.
This is $\boxed{\text{non-linear}}$.

(5) Does this table represent a linear function? If yes, find the function.

a)

x	0	1	2	3
$f(x)$	10	15	20	25

As x changes one unit
y changes (increases) 5 units
This rate of change is constant.

$m = 5$

$\boxed{\text{yes}}$

When $x=0$, the y-value is $10 = b$.

$\boxed{f(x) = 5x + 10}$

b)

x	-2	0	2	4
$f(x)$	6	3	0	-3

As x changes 2 units
y decreases 3 units
This rate of change is constant

$m = \frac{-3}{2}$

$\frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x}$

$\boxed{\text{yes}}$

when $x=0$, the y-value $3 = b$.

$\boxed{f(x) = \frac{-3}{2}x + 3}$

c)

x	0	1	2	3
$f(x)$	1	2	4	7

when x changes one unit
y sometimes changes 1 unit, 2 units, or 3 units.
The rate of change is not constant. $\boxed{\text{no}}$.

d)

x	-2	0	3	5
$f(x)$	7	7	7	7

No matter how x changes, y does not change.
This is a constant function (rate of change = 0)

$\boxed{f(x) = 7}$

$\boxed{\text{yes}}$

Midpoints

(6) a) Find the average of -3 and 5.

b) Plot $-3, 5$, and the average on a number line. What do we observe?

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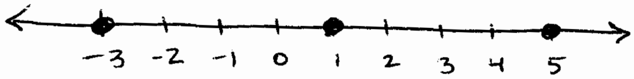
a) $\frac{-3+5}{2} = \frac{2}{2} = \boxed{1}$ average

b)



The average is exactly halfway between -3 and 5.

The average is the midpoint. $= \boxed{\frac{x_1+x_2}{2}. \text{ Midpoint formula}}$

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(7) a) Use graph paper and plot $(-2,1)$ and $(4,-3)$ on the x-y plane. (Neatly!)

b) Use the grid to draw the midpoint between $(-2,1)$ and $(4,-3)$. Identify the coordinates of the midpoint.

c) How would we calculate this midpoint?

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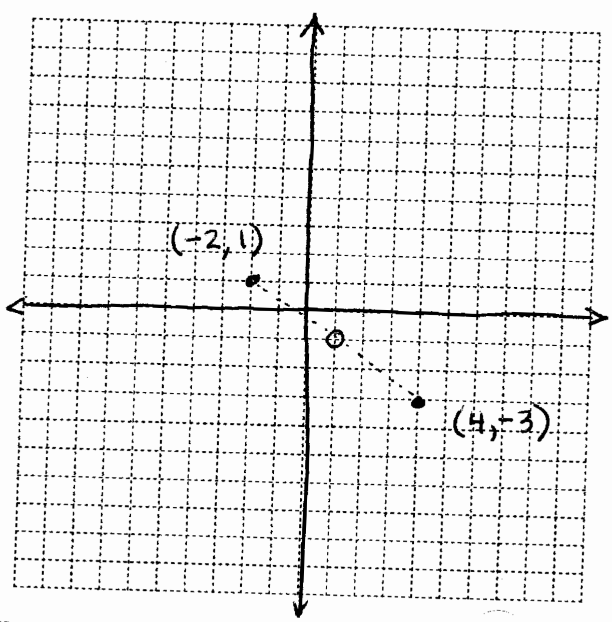
a)

b)



midpoint appears to be at $(1,-1)$

c) The x coordinate of the midpoint is the average of the x-coordinates.

The y-coordinate of the midpoint is the average of the y-coordinates.

$$\boxed{\text{Midpoint formula } \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)} = \left(\frac{-2+4}{2}, \frac{1+(-3)}{2}\right) = \left(\frac{2}{2}, \frac{-2}{2}\right) = (1,-1)$$

Examples

2.3 (1) Find the slope of a line passing through $(5, 2.5)$ and $(-.5, 3)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$ slope formula * memorize

Method 1: if $(x_1, y_1) = (5, 2.5)$ and $(x_2, y_2) = (-0.5, 3)$

$$m = \frac{3 - 2.5}{-.5 - 5} = \frac{.5}{-5.5} = \boxed{\frac{-1}{11}}$$

Method 2: if $(x_1, y_1) = (-.5, 3)$ and $(x_2, y_2) = (5, 2.5)$

$$m = \frac{2.5 - 3}{5 - (-.5)} = \frac{-.5}{5.5} = \boxed{\frac{-1}{11}}$$

How you decide which pair is subscript 2 is irrelevant, but be consistent for both Δx and Δy.

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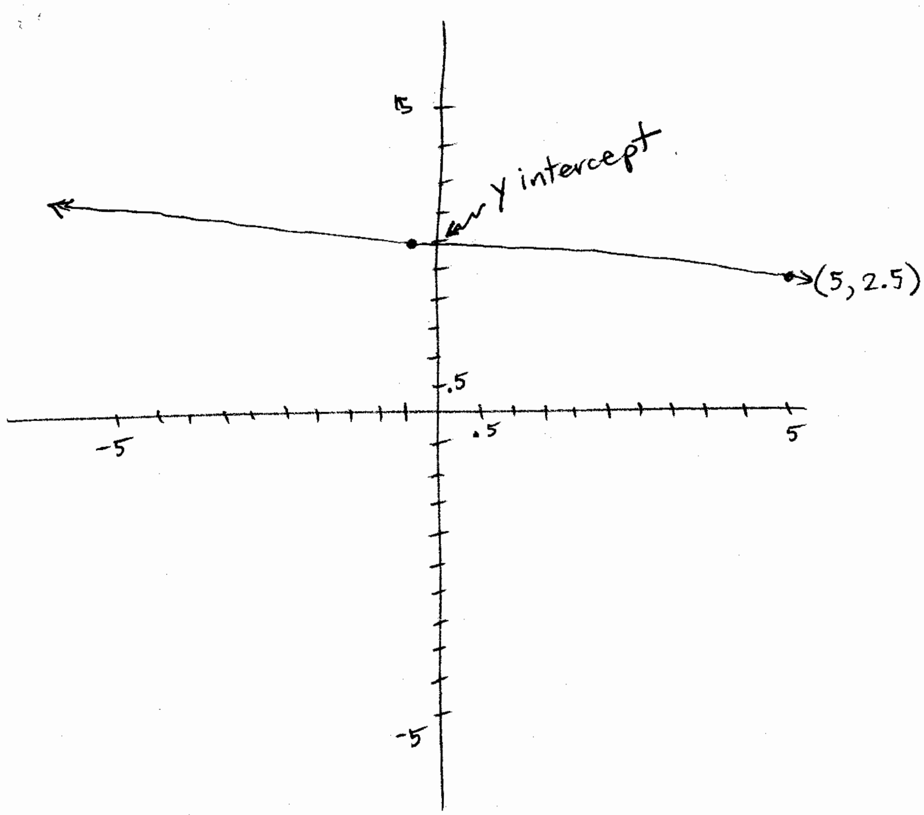
(2) Sketch the graph of the line in (1)



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(3) Interpret the slope in (2). negative = decrease

"The y-value decreases one unit when the x-value increases 11 units." OR "The y value decreases $\frac{1}{11}$ unit per x unit."

The x-intercept is a point (ordered pair) on the line, where the graph of the line crosses the x-axis.

The y-intercept is a point (ordered pair) on the line, where the graph of the line crosses the y-axis.

(4) Find the y-intercept of line in (1)-(2)(-3)

The y-value decreases $\frac{1}{11}$ unit per 1 unit on x

But...

The y-value decreases $\underline{?}$ unit per $\frac{1}{2}$ unit on x.

Method 1: Take $\frac{1}{2}$ of $\frac{1}{11}$ to find the change in y.

$\frac{1}{2} \cdot \frac{1}{11} = \frac{1}{22}$. The y value decreases $\frac{1}{22}$ unit

Subtract $\frac{1}{22}$ from y coordinate 3 to get y-coordinate.

$3 - \frac{1}{22} = \frac{66}{22} - \frac{1}{22} = \frac{65}{22}$ = new y-coordinate.

y intercept is $\boxed{\left(0, \frac{65}{22}\right)}$ or $\boxed{\left(0, 2.9\overline{54}\right)}$

If using decimal, the repeat bar is required, and only on 54, not 9.

Method 2: Substitute (-.5, 3) and (0, y) and $m = -\frac{1}{11}$ into slope formula and solve for y.

$$\frac{Y_2 - Y_1}{X_2 - X_1} = m$$

$$\frac{Y - 3}{0 - (-.5)} = \frac{-1}{11}$$

$$\frac{Y - 3}{.5} = \frac{-1}{11}$$

A ratio: fraction = fraction

cross-multiply

$11(y-3) = -.5$

$11y - 33 = -.5$

$11y = 33 - .5$

$11y = 32.5$

$y = \frac{32.5}{11}$

$y = 2.9\overline{54} = \frac{65}{22}$

$$\boxed{\text{The y-intercept is } (0, 2.9\overline{54}) \text{ or } \left(0, \frac{65}{22}\right)}$$

(5) Find the equation of the line in (1) - (4).

$y = mx + b$ is called the slope-intercept form of the equation of a line where $m =$ slope $b =$ y-coordinate of y-intercept

substitute $m = \frac{-1}{11}$ from (1)

substitute $b = \frac{65}{22}$ from (4)

$$\boxed{y = \frac{-1}{11}x + \frac{65}{22}}$$

This could also be written

$y = \frac{65}{22} - \frac{1}{11} \cdot x$

start at $\frac{65}{11}$ change the y coordinate $\frac{1}{-11}$ times x.

⑥ Find the x-intercept of the line in ① - ⑤

To cross the x-axis, the y-coordinate must be 0.
Set $y=0$ in the equation of the line.

$$y = -\frac{1}{11}x + \frac{65}{22}$$

$$0 = -\frac{1}{11}x + \frac{65}{22}$$ subtract $\frac{65}{22}$ both sides

$$-\frac{65}{22} = -\frac{1}{11}x$$ cross-mult

$$(-65)(11) = -22x$$ div both sides by -22

$$\frac{(-65)(11)}{(-22)} = x$$ reduce $\frac{11}{22} = \frac{1}{2}$

$$\frac{+65}{+2} = x$$ reduce $\frac{(-)}{(-)} = +$

$$\frac{65}{2} = x$$

The x intercept is $\boxed{\left(\frac{65}{2}, 0\right)}$ or $\boxed{(32.5, 0)}$

In general:

> To find <u>x-intercept(s)</u> of a graph
> subst $y=0$ into equation
>
> To find <u>y-intercept(s)</u> of a graph
> subst $x=0$ into equation.

works for <u>any</u> equation. Not just lines. ☺